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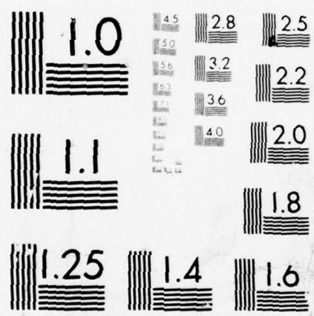

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Cambridge, MA 02139

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POTENTIAL THEORY OF STEADY MOTION OF SHIPS, PART 3: WAVE RESISTANCE

by

Francis Noblesse

November 1978

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POTENTIAL THEORY OF STEADY MOTION OF SHIPS,

PART 3: WAVE RESISTANCE

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ABSTRACT

New expressions for the wave resistance of a ship in steady rectilinear motion in a calm sea are presented. Specifically, a sequence of iterative approximations R_0, R_1, R_2, ... is defined. Of particular interest are the zeroth approximation R_0 and the first approximation R_1. The zeroth approximation R_0 provides a new simple explicit wave-resistance formula which may be regarded as a generalization of the classical formulas proposed by Michell in 1898 and Hogner in 1932. A noteworthy feature of the approximation R_0 is that it involves a line integral along the ship waterline, which causes a drastic reduction in the wave resistance at low Froude number and is particularly significant for blunt ship forms. Although the first approximation R_1 is of course more complex than the zeroth approximation R_0, it provides a fairly simple explicit approximation to the wave resistance which is more refined than the approximation R_0 and may be of greater usefulness for practical purposes. The first approximation R_1 is indeed regarded as the main result of this study. Comparison between the zeroth approximation R_0 and the classical Michell approximation for a wedge-like ship bow form suggests that the present theory may remedy to some of the typical discrepancies between experimental and theoretical wave-resistance curves.

INTRODUCTION

The single most important goal of a theory of steady rectilinear motion of a ship in a calm sea no doubt is the prediction of the drag, so-called wave resistance, experienced by a ship as a result of the surface gravity waves it creates. Exact expressions (to be sure, within the limitations of potential-flow theory) and related explicit approximations for the wave resistance of a ship are presented in this study, which corresponds to Part 3 of the theory of steady motion of ships developed in Noblesse [1]. Only the case of displacement ships is considered here for shortness; however, modification of the analysis and of the final formulas for the case of other types of ships, e.g. fully-submerged bodies, multihull vessels, and surface-effect ships, is straightforward (problems associated with lift and cavitation for hydrofoils, and planing effects and spray formation for fast boats, are not considered in the present theory however).

The wave resistance is defined by a set of three equations, namely, (i) the classical Havelock wave-resistance formula (9) expressing the wave resistance R in terms of the Kochin free-wave spectrum function $\Omega(\theta)$, which is directly related to the free-wave pattern trailing far behind the ship, (ii) formula (7) defining the Kochin spectrum function $\Omega(\theta)$ in terms of the velocity potential ϕ of the disturbance flow caused by the ship in its "near field", and (iii) the integral equation (1) for determining the velocity potential ϕ (this integral equation was derived in Part 2 of [1]). Actually, to these three equations one should add equations for determining the hydrodynamic lift and moment, and the resulting sinkage and trim, experienced by the ship; however, these additional equations are not considered explicitly in this study.

In practice, the integral equation (1), and formulas (7) and (9) may be used for defining a sequence of iterative approximations to the wave resistance. Specifically, corresponding to the successive iterative approximations $\phi_0 \equiv 0$, ϕ_I, ϕ_2, ... to the solution ϕ of the integral equation (1), we may readily associate the approximations Ω_0, Ω_1, Ω_2, ... to the Kochin free-wave spectrum function $\Omega(\theta)$, and the approximations R_0, R_1, R_2, ... to the wave resistance R, by using formulas (7) and (9) in which we need only replace ϕ by ϕ_k, Ω by Ω_k, and R by R_k, with k = 0, 1, 2, ... ; the iterative approximations ϕ_I and ϕ_2 to the solution of the integral equation (1) were obtained previously in Part 2 of [1], and are given in this Part 3 by formulas (2) and (3). The approximations Ω_0, Ω_1, and Ω_2 are derived explicitly, and discussed in some detail. The general recurrence relations defining the iterative approximations ϕ_k and Ω_k are also given [specifically by formulas (30) and (31)]. Of main interest for practical purposes, however, are the "zeroth approximation" Ω_0 and the first approximation Ω_1 corresponding to the approximations $\phi_0 \equiv 0$ and ϕ_I, respectively.

The zeroth approximation $\Omega_0(\theta)$, given by formula (10), provides a new simple explicit wave-resistance formula, which may be regarded as a generalization of the classical formulas proposed by Michell in 1898 and Hogner in 1932. Indeed, these classical approximations are obtained in the present study as particular limiting cases of the zeroth approximation Ω_0. A noteworthy feature of the approximation Ω_0 is that it involves a line integral along the ship waterline, as it may readily be seen from formula (10). This line integral is shown to cause a drastic reduction in the value of the wave resistance at low Froude number, and is particularly significant for blunt ship forms.

The first approximation $\Omega_1(\theta)$, defined by formula (22), is associated with the approximation ϕ_I given by formula (2). Although this first approximation Ω_1 is obviously more complex than the zeroth approximation Ω_0, it provides a fairly simple explicit approximation to the Kochin spectrum function $\Omega(\theta)$ which clearly is more refined than the approximation Ω_0, and may be of greater usefulness for practical purposes. In particular, the first approximation Ω_1 incorporates correction terms associated with the hull boundary condition, including effects of sinkage and trim, and corrections for free-surface nonlinearities. For practical purposes, the first approximation is regarded as the main result of the present study. Various ways of improving, and simplifying, this approximation are briefly discussed.

Finally, preliminary numerical results are presented. Specifically, a comparison between the zeroth approximation R_0 and the classical Michell approximation for a wedge-like ship bow form is made. The results, shown in figure 2, indicate that — compared with the Michell approximation (M) — the new approximation R_0 yields a significant overall reduction in the value of the wave resistance, a very appreciable reduction in the magnitude of the oscillations (humps and hollows) in the wave resistance curve, and a notable phase shift of these oscillations towards lower values of the Froude number. These encouraging preliminary results suggest that the present theory may thus remedy to some of the typical discrepancies between experimental and theoretical wave resistance curves.

1. The integral equation for the velocity potential ϕ, and the approximations ϕ_I and ϕ_2

We consider a displacement ship in steady, rectilinear motion at the free surface of an otherwise calm sea assumed to be of infinite depth and lateral extent. Water is supposed to be homogeneous and incompressible. Surface tension is neglected. Irrotational flow is assumed; however, ad hoc corrections for the viscous boundary layer and wake around and behind the ship are incorporated into the present potential-flow model, in the manner explained in [1]. Ad hoc corrections for effects of spray formation along the waterline, and for effects of wavebreaking are also included, as in [1].

A moving system of coordinates attached to the ship is chosen, so that the flow is independent of time. The z axis is taken vertical, positive upwards, with the undisturbed free surface taken as the plane $z = 0$. The x axis is parallel to the direction of motion of the ship and positive towards the ship stern. Flow variables are rendered dimensionless with respect to the speed of the ship U, the acceleration of gravity g, and the fluid density ρ, as it is shown in equations (1.5) in [1].

The hydrodynamical problem amounts to determining the (dimensionless) velocity potential ϕ of the disturbance flow caused by the ship. This problem was formulated in [1] as a "generalized Neumann-Kelvin problem" in a "solution domain" (d) bounded by some "fictitious hull surface" (h), which may (but need not) be taken as the submerged hull of the ship in position of rest [in principle, the "fictitious hull surface" (h) may be chosen arbitrarily], and the undisturbed free surface (f), which is the portion of the plane $z = 0$ outside the intersection curve (c) of the surface (h) with the plane $z = 0$. It is shown in [1] that the "generalized Neumann-Kelvin problem" can be formulated in "integral form", given by the integral equation (2.21). This integral equation is given here for easy reference:

$$\boxed{\begin{aligned}\phi_0 = {} & \int_h G\nu da - \oint_c G\nu^2\mu ds + \int_h (\phi-\phi_0)G_n da + \oint_c [G(\sigma\phi_s+\tau\phi_t)-(\phi-\phi_0)G_x]\mu ds \; + \\ & + \int_f G(q_{nl}+q_{wb})dxdy + \int_h G(q_{hf}+q_{bl})da - \oint_c G[(q_{hf}+q_{bl})\nu\mu-q_s]ds + \int_d Gq_w dv\end{aligned}} \quad , \quad (1)$$

where the significance of the various undefined symbols will now be explained.

The symbol ϕ_0 is meant for $\phi(\vec{x}_0)$ where $\vec{x}_0$ is an arbitrary point in the solution domain (d) including its boundary (h) + (f) + (c), while ϕ is meant for $\phi(\vec{x})$ where $\vec{x}$ represents the "point of integration" (integration variable) in the above integrals; the point $\vec{x}_0$ thus is the "field point" where the potential is being

evaluated, while $\vec{x}$ represents the "dummy" variable of integration. The function $G \equiv G(\vec{x}_0,\vec{x})$ is the fundamental solution (Green function) appropriate for the problem. Specifically, the function $G(\vec{x}_0,\vec{x})$ represents the (dimensionless) linearized velocity potential of the disturbance flow caused at point $\vec{x}_0(x_0,y_0,z_0\leq 0)$ by a unit "outflow" at point $\vec{x}(x,y,z\leq 0)$, associated with a submerged source if $z < 0$ or a flux across the free surface if $z = 0$, in an oncoming uniform stream along the positive x axis. Simplified new expressions for the fundamental function G and its gradient ∇G may be found in Noblesse [2].

The symbol da in the three surface integrals over (h) represents the differential element of area of (h), while ds in the three line integrals around the "waterline" (c) represents the differential element of arc length of (c), and dv in the volume integral over the solution domain (d) is the differential element of volume. In the first integral, we have $\nu \equiv \nu(\vec{x}) \equiv \vec{n}(\vec{x})\cdot\vec{i}$, where $\vec{n}(\vec{x})$ is the unit inward (that is, $\vec{n}$ is pointing towards the interior of the ship) normal vector to (h) at point $\vec{x}$ of (h), and $\vec{i}$ is the unit positive vector along the x axis. In the line integrals around (c), we have $\nu \equiv \nu(s) \equiv \vec{n}(s)\cdot\vec{i}$ where $\vec{n}(s)$ is the normal to (h) at point s of (c), while μ is defined as $\mu \equiv \mu(s) \equiv \vec{n}'(s)\cdot\vec{i}$, where $\vec{n}'$ is the unit inward normal vector to (c) in the plane $z = 0$; in the (fairly common) case when the surface (h) intersects the plane $z = 0$ orthogonally, we have $\vec{n}(s) \equiv \vec{n}'(s)$ and $\nu(s) \equiv \mu(s)$. In the second line integral around (c), the symbols σ and τ are defined as $\sigma \equiv \vec{s}\cdot\vec{i}$ and $\tau \equiv \vec{t}\cdot\vec{i}$, where $\vec{s} \equiv \vec{s}(s)$ is the unit tangent vector, at point s, to the "waterline" (c) oriented in the counterclockwise direction in the (x,y) plane, and $\vec{t} \equiv \vec{t}(s)$ is the unit vector tangent to (h), mutually orthogonal to $\vec{s}(s)$ and the normal $\vec{n}(s)$ to (h) at point s of (c), and pointing downwards. In this line integral, the notation $\phi_s \equiv \partial\phi(s,t,n)/\partial s$, $\phi_t \equiv \partial\phi(s,t,n)/\partial t$, and $G_x \equiv \partial G(\vec{x}_0,\vec{x})/\partial x$ was used for shortness. The usual notation $G_n \equiv \nabla G(\vec{x}_0,\vec{x})\cdot\vec{n}(\vec{x})$ was also used in the third integral. It will be noted that the axes x, y, and z, the "fictitious hull surface" (h), the undisturbed free surface (f), the "waterline" (c), the elements of area da and of arc length ds, and the unit vectors, $\vec{i}$, $\vec{n}$, $\vec{n}'$, $\vec{s}$, and $\vec{t}$, are shown in figure 1.

It remains to define the terms q_{nl}, q_{hf}, q_s, q_{bl}, q_{wb}, and q_w. The term q_{nl} in the integral over the undisturbed free surface (f) represents the "NonLinear free-surface correction flux", which accounts for the nonlinear terms in the free-surface boundary condition and for the difference in position between the actual free surface and the plane $z = 0$ of the undisturbed free surface where the free-surface condition is enforced for mathematical simplicity; the "nonlinear free-surface flux" q_{nl} is given by

$$q_{nl} = \left[\phi_z + \phi_{xx} + \left(|\nabla\phi|^2\right)_x + \tfrac{1}{2}\nabla\phi\cdot\nabla|\nabla\phi|^2\right]_{z=-\phi_x-\frac{1}{2}|\nabla\phi|^2} - \left[\phi_z + \phi_{xx}\right]_{z=0} \quad . \tag{1a}$$

The term q_{hf} is meant for the "Hull-Form correction flux" associated with the fact that the "fictitious hull surface" (h) may be different from the actual ship hull surface (H); the "hull-form correction flux" q_{hf} is given by

$$q_{hf} = (\vec{i} + \nabla\phi)_H \cdot \vec{N} - (\vec{i} + \nabla\phi)_h \cdot \vec{n} \ , \tag{1b}$$

where $\vec{N}$ is the unit inward normal vector to (H), and the notation $(\ )_H$ and $(\ )_h$ means that the expression between the parentheses, namely $\nabla\phi$, is to be evaluated on (H) and (h), respectively. Finally, the terms q_{bl}, q_s, q_{wb}, and q_w are ad hoc corrections associated with the viscous Boundary Layer around the ship hull, Spray formation along the waterline, WaveBreaking at the free surface, and the viscous Wake trailing behind the ship, respectively.

It may be useful to emphasize that the integral equation (1) is valid for $\vec{x}_0$ in the solution domain (d) including its boundary (h) + (f) + (c), as it was already noted, so that this equation defines the disturbance velocity potential ϕ everywhere in (d) + (h) + (f) + (c). The solution ϕ of the integral equation (1) may be determined in practice by using an iterative method of solution, as it is explained in [1].

Briefly, if we merely ignore all the unknown terms in the integral equation (1), we obtain the <u>initial approximation</u> $\phi_I(\vec{x}_0)$, which is thus given by

$$\boxed{\phi_I(\vec{x}_0) = \int_h G(\vec{x}_0,\vec{x})\nu(\vec{x})da(\vec{x}) - \oint_c G(\vec{x}_0,x,y,0)\nu^2(s)\mu(s)ds} \quad . \tag{2}$$

A <u>second approximation</u> $\phi_2(\vec{x}_0)$ can then be determined by evaluating the previously-ignored unknown terms (however, the correction terms for "real-fluid effects" q_{bl}, q_s, q_{wb}, and q_w are neglected in the following second approximation ϕ_2) on the basis of the initial approximation ϕ_I to ϕ. After some manipulations, we may obtain

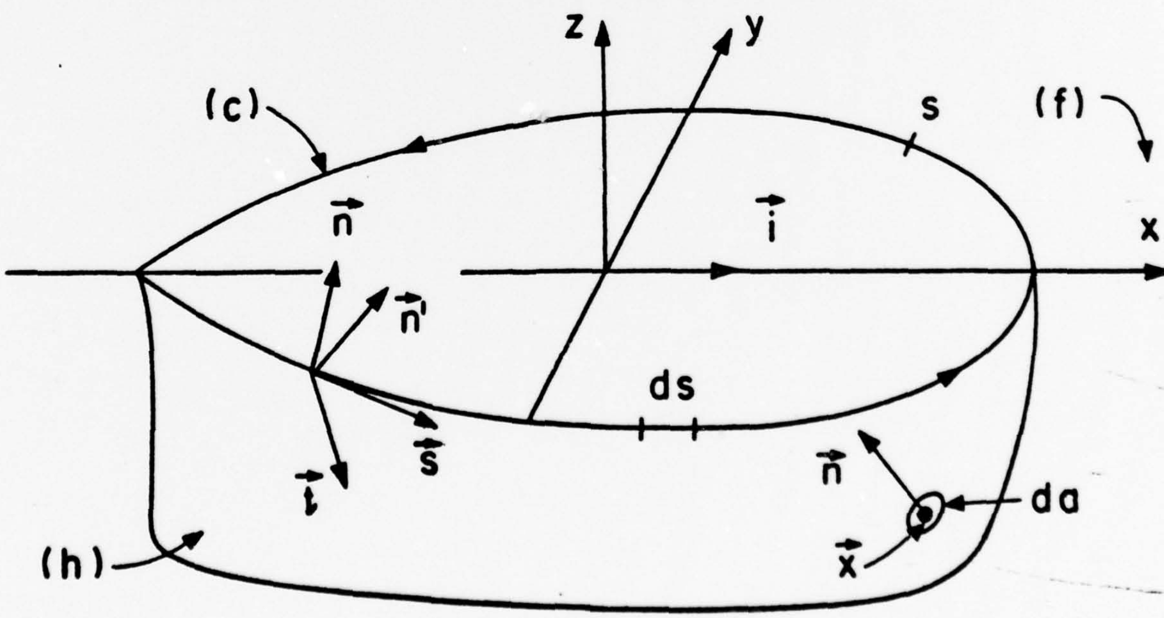
$$\boxed{\phi_2(\vec{x}_0) = \phi_I(\vec{x}_0) + \int_h G(\nu+\phi_n^I)_H da - \oint_c G(\nu+\phi_n^I)_H \nu\mu ds + \int_f G q_{nl}^I dxdy} \quad , \tag{3}$$

where the notation $(\nu+\phi_n^I)_H$ is meant for $(\nu)_H + (\phi_n^I)_H \equiv \vec{i}\cdot\vec{N} + (\nabla\phi_I)_H\cdot\vec{N} \equiv (\vec{i}+\nabla\phi_I)_H\cdot\vec{N}$, which represents the fluid flux across the actual ship hull (H) in the initial approximation $\vec{i} + \nabla\phi_I$, and the nonlinear free-surface flux q_{nl}^I is given by

$$q^I_{n1} = \left(|\nabla\phi_I|^2\right)_x - \phi^I_x(\phi^I_z + \phi^I_{xx})_z \quad , \tag{3a}$$

While higher iterative approximations ϕ_k, $k \geq 3$, could readily be defined in principle, numerical evaluation of these higher approximations would probably be too considerable a task in practice. However, it seems reasonable to hope that the second approximation ϕ_2, or even perhaps the initial approximation ϕ_I, may be sufficiently accurate for most practical applications. Further discussion of the above approximations ϕ_I and ϕ_2 may be found in [1], where variations about these iterative approximations are also examined. It may finally be noted here that the value of the potential ϕ in any portion of the fluid domain (D) outside the solution domain (d), e.g. in the region between the actual free surface (F) and the plane z = 0 wherever (F) lies above the plane z = 0, may be determined from the value of ϕ in (d) by means of analytical continuation.

Figure 1: Definition sketch



2. The Kochin free-wave spectrum function $\Omega(\theta)$ and the Havelock wave-resistance formula

A ship's wave resistance is related in a simple manner to the wave pattern trailing behind the ship, as it is well known since Havelock [3]. The disturbance velocity potential ϕ in the "far field", that is at a large distance away from the ship, may be expressed in terms of the value of ϕ in the "near field", that is on the hull surface (h) + (c) and on the free surface (f) in the vicinity of the ship, by means of the equation

$$\phi_0 = \int_h G\nu da - \oint_c G\nu^2\mu ds + \int_h \phi G_n da + \oint_c [G(\sigma\phi_s+\tau\phi_t)-\phi G_x]\mu ds + \int_f G(q_{nl}+q_{wb})dxdy +$$
$$+ \int_h G(q_{hf}+q_{bl})da - \oint_c G[(q_{hf}+q_{bl})\nu\mu - q_s]ds + \int_d Gq_w dv \;, \qquad (4)$$

which may be derived from the integral equation (1) by replacing the term $\phi - \phi_0$ in the third and fourth integrals by ϕ since we have $\phi_0 \to 0$ as $|\vec{x}_0| \to \infty$, and therefore $|\phi_0| << |\phi|$ for $\vec{x}_0$ in the far field and $\vec{x}$ in the near field. It is interesting to note here in passing that while expression (4) for the disturbance velocity potential $\phi(\vec{x}_0)$ in the far field was obtained above as the "far-field limit" of the integral equation (1), it is actually valid also for $\vec{x}_0$ in the near field, provided only that $\vec{x}_0$ is strictly outside the hull surface (h) + (c), that is equation (4) holds for $\vec{x}_0$ in the domain (d) + (f) - (h) - (c); indeed, equation (4) corresponds to equation (2.11a) in [1].

Far behind the ship, that is for $x_0 >> 1$, the foregoing expression for ϕ_0 can actually be greatly simplified by replacing the Green function $G(\vec{x}_0,\vec{x})$ by the well-known asymptotic approximation

$$G \sim \mathrm{Re}\, \frac{i}{\pi}\int_{-\pi/2}^{\pi/2} e^{\{(z_0+z)+i[(x_0-x)\cos\theta+(y_0-y)\sin\theta]\}\sec^2\theta} \sec^2\theta d\theta \quad \text{as } x_0 \to \infty \qquad (5)$$

which may be obtained, for instance, from equations (19), (23) and (10c) in Noblesse [4]; the above approximation differs from the exact expression for G by terms associated with a near-field disturbance which "dies out" like $1/x_0$ as $x_0 \to \infty$, and may be discarded inasmuch as we are only interested in the trailing wave pattern far behind the ship.

By using the asymptotic approximation (5) into equation (4), we may express the disturbance potential $\phi(\vec{x}_0)$ far behind the ship in the form

$$\phi(\vec{x}_0) \sim \mathrm{Re}\, \frac{i}{\pi} \int_{-\pi/2}^{\pi/2} \Omega(\theta) \sec^2\theta \; e^{[z_0+i(x_0\cos\theta+y_0\sin\theta)]\sec^2\theta} \, d\theta \quad \text{as } x_0 \to \infty \quad , \tag{6}$$

where the function $\Omega(\theta)$ is given by

$$\boxed{\begin{aligned} \Omega(\theta) = & \int_h \mathcal{E}\nu da - \oint_c \mathcal{E}^0 \nu^2 \mu ds + \int_h \phi \mathcal{E}_n da + \oint_c [\mathcal{E}^0(\sigma\phi_s + \tau\phi_t) - \phi \mathcal{E}^0_x]\mu ds + \\ & + \int_f \mathcal{E}^0(q_{n1}+q_{wb}) dxdy + \int_h \mathcal{E}(q_{hf}+q_{b1}) da - \oint_c \mathcal{E}^0[(q_{hf}+q_{b1})\nu\mu - q_s] ds + \int_d \mathcal{E} q_w dv \end{aligned}} \quad , \tag{7}$$

with the functions $\mathcal{E} \equiv \mathcal{E}(\theta;\vec{x})$ and $\mathcal{E}^0 \equiv \mathcal{E}(\theta;x,y,z=0)$ defined as

$$\underline{\mathcal{E} = e^{[z-i(x\cos\theta+y\sin\theta)]\sec^2\theta}} \quad , \quad \underline{\mathcal{E}^0 = e^{-i(x\cos\theta+y\sin\theta)\sec^2\theta}} \quad . \tag{7a,b}$$

The equation of the free surface far behind the ship is given by $z_0 = -\phi_{x_0}(x_0,y_0,z_0=0)$ since the nonlinear terms in the free-surface boundary condition may be neglected at a sufficiently large distance away from the ship. By using equation (6) we may then obtain

$$z_0 \sim \mathrm{Re}\, \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} \Omega(\theta) \sec^3\theta \; e^{i(x_0\cos\theta+y_0\sin\theta)\sec^2\theta} \, d\theta \quad \text{as } x_0 \to \infty \; . \tag{8}$$

Equations (6) and (8) express the potential $\phi(\vec{x}_0)$ and the equation of the free surface $z_0(x_0,y_0)$ far downstream from the ship in terms of a familiar superposition of elementary plane waves with amplitude $\Omega(\theta)\sec^3\theta$. The function $\Omega(\theta)$ corresponds to a particular case of the well-known function introduced by Kochin for determining the drag, lift, and moment acting upon a ship, which is often referred to as the "free-wave spectrum" in the literature on "wave analysis" (see for instance Eggers, Sharma, and Ward [5]); the function $\Omega(\theta)$ defined by equation (7,a,b) will be referred to as the "Kochin free-wave spectrum function" in the present study.

The (dimensionless) wave resistance, R say ($R \equiv R'g^2/\rho U^6$, where R' is dimensional), may be directly determined from the "Kochin free-wave spectrum function" $\Omega(\theta)$ by means of the well-known "Havelock wave-resistance formula"

$$\boxed{R = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} |\Omega(\theta)|^2 \sec^3\theta d\theta} \quad , \tag{9}$$

which is given for instance in [5] equation (12) p. 118. In the usual case of a ship which is symmetric with respect to its centerplane $y = 0$, the function $|\Omega(\theta)|$ is even, so that the "Havelock wave-resistance formula" (9) may be expressed in the form

$$R = \frac{1}{\pi}\int_{0}^{\pi/2} |\Omega(\theta)|^2 \sec^3\theta d\theta \quad . \tag{9a}$$

The "Havelock wave resistance formula" (9) for the wave resistance R, formula (7) for the "Kochin free-wave spectrum function" $\Omega(\theta)$, and the integral equation (1) for determining the disturbance velocity potential ϕ in the "near field" of the ship, form a set of three equations for determining the wave resistance of a ship in steady motion. Actually, to these three equations we must also add equations for determining the sinkage and trim experienced by the ship, which requires evaluation of the hydrodynamic lift and moment acting upon the ship. However, the equations for lift and moment and sinkage and trim will not be considered explicitly in the present study, which is mainly concerned with the "hydrodynamical problem" defined in Part 1 of [1], that is the problem of predicting the flow caused by a ship, and in this Part 3 the wave resistance, assuming the shape, and position, of the ship to be given.

The integral equation (1) and formulas (7) and (9) are essentially "exact", that is within the limitations of the present potential-flow model, and in principle these equations thus provide a basis for obtaining an essentially "exact" value of the wave resistance of a ship in steady motion. In practice, equations (1), (7), and (9) may be used for determining a sequence of "iterative approximations" to the wave resistance R: specifically, corresponding to the successive iterative approximations $\phi_0 \equiv 0$, ϕ_I, ϕ_2, ... to the solution ϕ of the integral equation (1), we may readily associate the approximations Ω_0, Ω_1, Ω_2, ... to the Kochin free-wave spectrum function $\Omega(\theta)$, and the approximations R_0, R_1, R_2, ... to the wave resistance R, by using formulas (7) and (9) in which we need only replace ϕ by ϕ_k, Ω by Ω_k, and R by R_k, with $k = 0, 1, 2, \ldots$. The approximations Ω_0, Ω_1, and Ω_2 are given and discussed in some detail in the following three sections; several particular limiting cases of these approximations are also discussed.

3. Elementary wave-resistance formulas: the zeroth approximation R_0 and particular limiting cases

In this section, five elementary wave resistance approximations are derived and discussed. These are the "zeroth approximation" R_0, and its four particular limiting cases the Hogner "fine-ship approximation" R_H, the Hogner "flat-ship approximation" R_H^f, the Michell "thin-ship approximation" R_M, and the Maruo-Tuck-Vossers "slender-ship approximation" R_{MTV}.

The simplest, and no doubt the crudest, possible approximation to the "Kochin free-wave spectrum function" $\Omega(\theta)$, and hence to the wave resistance R, is obtained if one merely ignores the integral equation (1) for determining the disturbance velocity potential ϕ in the "near field", and puts $\phi = 0$ in formula (7) for $\Omega(\theta)$. This approximation may perhaps be regarded as a "slender-ship approximation" since it can be expected to be the better the smaller the disturbance potential ϕ, that is the more "slender" the ship; however, it will simply be referred to as the "zeroth approximation" in the present study. This "zeroth approximation" thus is given by

$$\boxed{\begin{aligned}\Omega_0(\theta) = &\int_h e^{[z - i(x\cos\theta + y\sin\theta)]\sec^2\theta}\,\nu(\vec{x})da(\vec{x}) \\ &- \oint_c e^{-i(x\cos\theta + y\sin\theta)\sec^2\theta}\,\nu^2(s)\mu(s)ds\end{aligned}} \quad , \tag{10}$$

as it may readily be obtained from formulas (7,a,b).

If the ship is sufficiently "fine", that is if the angle between the "waterline" (c) and the x axis is sufficiently small, we have $\nu^2|\mu|<<|\nu|<<1$ and the line integral in formula (10) may be neglected in comparison with the surface integral. It is interesting that the approximation, $\Omega_H(\theta)$ say, defined by this surface integral, that is

$$\Omega_H(\theta) = \int_h e^{[z - i(x\cos\theta + y\sin\theta)]\sec^2\theta}\,\nu(\vec{x})da(\vec{x}) \tag{11}$$

in fact corresponds to the approximation proposed by Hogner [6] in 1932; indeed, the wave resistance approximation R_H obtained by replacing $\Omega(\theta)$ by $\Omega_H(\theta)$ in formula (9) is identical to Hogner's wave resistance formula. The approximation $\Omega_H(\theta)$, R_H will be referred to as the Hogner "fine-ship approximation", or simply as the Hogner approximation.

If the equation of the (fictitious) hull surface (h) may be written in the form $y = \pm\, b(x,z)$, where $b(\equiv gB/U^2)$ is dimensionless like the other flow variables, and the points (x,z) belong to the projection (h_y) of the surface (h) onto the ship centerplane $y = 0$, the surface integral in formula (11) may be transformed into the following double integral

$$\Omega_H(\theta) = 2 \iint_{h_y} e^{z\sec^2\theta - ix\sec\theta} \cos[b(x,z)\tan\theta\sec\theta]\, b_x(x,z)\,dx\,dz \quad . \tag{11a}$$

If the ship is "thin", that is if $b(x,z)$ is sufficiently small that the term $\cos[b(x,z)\tan\theta\sec\theta]$ may be approximated by 1, the Hogner "fine-ship approximation" $\Omega_H(\theta)$ given in equation (11a) becomes the famous <u>Michell "thin-ship approximation"</u> $\Omega_M(\theta)$ say, first obtained by Michell in 1898 and given by

$$\Omega_M(\theta) = 2 \iint_{h_y} e^{z\sec^2\theta - ix\sec\theta}\, b_x(x,z)\,dx\,dz \quad . \tag{12}$$

Several differences between the Hogner approximation Ω_H, R_H and the Michell approximation Ω_M, R_M are readily apparent and may be noted here. An interesting difference between $\Omega_H(\theta)$ and $\Omega_M(\theta)$ is that while the Michell approximation $\Omega_M(\theta)$ is proportional to the ship's beam B, and R_M therefore is proportional to B^2 (as it is very well known), the Hogner approximation $\Omega_H(\theta)$ clearly is not proportional to B (although Ω_H becomes proportional to B in the limit $B \to 0$). It can also readily be seen that differences between R_H and R_M may be expected to be the larger the bigger the beam and the smaller the Froude number; this is due to the fact that we have $b = gB/U^2 = (B/L)(gL/U^2) = \varepsilon/F^2$, where $\varepsilon \equiv B/L$ is the "geometrical thinness" ratio, and $F \equiv U/(gL)^{1/2}$ is the Froude number based on the ship length L. It is interesting to note that the "thin-ship assumption" $b \ll 1$ used in deriving expression (12) for the Michell approximation Ω_M from expression (11a) for the Hogner approximation Ω_H implies not only "<u>geometrical thinness</u>", characterized by $\varepsilon \equiv B/L \ll 1$, but also <u>"Froude thinness"</u>, that is $\varepsilon/F^2 \ll 1$. It may also be interesting to note that while most ships would seem to be "geometrically thin", in that ε for most ships is fairly small compared with 1, they are usually not "Froude thin", in that the value of the ratio ε/F^2 is commonly equal to 1 or 2, which hardly seems small enough to justify approximating the term $\cos[b(x,z)\tan\theta\sec\theta]$ by 1. However, this approximation is certainly justified for small values of θ, corresponding to the "transverse waves" in the free-wave spectrum; on the other hand, the approximation $b\tan\theta\sec\theta \ll 1$ clearly

is not valid for values of $|\theta|$ close to $\pi/2$, that is for the "divergent waves" in the wave spectrum. Two factors which contribute to reducing the differences between R_H and R_M may finally be noted: these factors are that the term $b(x,z)\tan\theta\sec\theta$ is small for $b(x,z)$ small, that is at the ship bow and stern (which, however, are the main contributors to the wave resistance), and for θ small, that is for the part of the wave resistance corresponding to the "transverse waves" (which are known, however, to account for a significant portion of the wave resistance).

If the equation of the (fictitious) hull surface (h) is expressed in the form $z = -d(x,y)$, where $d(\equiv gD/U^2)$ is dimensionless, and the points (x,y) belong to the projection, (h_z) say, of the surface (h) onto the "waterplane" $z = 0$, the surface integral (11) may be transformed into the following double integral

$$\Omega_H(\theta) = \iint_{h_z} e^{-[d(x,y) + i(x\cos\theta + y\sin\theta)]\sec^2\theta} d_x(x,y)\,dxdy \quad .$$

If the ship is "flat", that is if $d(x,y)$ is sufficiently small that the term $\exp[-d(x,y)\sec^2\theta]$ may be approximated by 1, we obtain the Hogner "flat-ship approximation", $\Omega_H^f(\theta)$ say, given by

$$\Omega_H^f(\theta) = \iint_{h_z} e^{-i(x\cos\theta + y\sin\theta)\sec^2\theta} d_x(x,y)\,dxdy \quad . \tag{13}$$

The approximation R_H^f to the wave resistance associated with the above approximation Ω_H^f to the Kochin free-wave spectrum function is identical to the "flat-ship wave resistance formula" proposed by Hogner [6] in 1932. The differences between the Hogner "fine-ship approximation" Ω_H and the Michell "thin-ship approximation" Ω_M discussed previously may also be observed — with evident modifications — between the Hogner approximations Ω_H and Ω_H^f. In particular, it will be noted that Ω_H^f is proportional to the draft D of the ship, and the wave resistance R_H^f therefore is proportional to D^2, which is not true for the approximation Ω_H, R_H. It will also be noted that the approximation $d(x,y)\sec^2\theta \ll 1$ must obviously break down as $|\theta| \to \pi/2$ and as the Froude number $F \to 0$.

If the ship is both "thin" and "flat", that is if the term $\exp(-iy\tan\theta\sec\theta)$ in formula (13) for the Hogner "flat-ship approximation" Ω_H^f and the term $\exp(z\sec^2\theta)$ in formula (12) for the Michell "thin-ship approximation" Ω_M can be approximated by 1, both approximations (12) and (13) yield the Maruo-Tuck-Vossers "slender-ship approximation"

$$\Omega_{MTV}(\theta) = \int_{bow}^{stern} e^{-ix\sec\theta} \lambda(x)dx \quad , \tag{14}$$

where $\lambda(x)$ is given by

$$\lambda(x) = \int_{-b(x)}^{+b(x)} d_x(x,y)dy = 2\int_{-d(x)}^{0} b_x(x,z)dz \quad , \tag{14a}$$

with d(x) and b(x) representing the local (at section x) draft and half beam of the ship, respectively. The above approximation Ω_{MTV} was obtained by Maruo [7], Tuck [8], and Vossers [9] in the early 1960's by using the method of "matched asymptotic expansions". It may readily be seen that the Maruo-Tuck-Vossers approximation Ω_{MTV} is proportional to the product BD of the ship's beam and draft, and the wave resistance R_{MTV} is proportional to B^2D^2. An appealing feature of the approximation R_{MTV} defined by formulas (14,a) and (9) resides in its remarkable simplicity. Unfortunately, this wave resistance approximation is practically useless for realistic ship hull forms. In particular, the source strength $\lambda(x)$ must vanish at the ship bow and stern for the wave resistance integral (9) merely to exist; this "existence condition" is extremely restrictive, and indeed is not satisfied by usual ship hull forms.

It may be interesting to note that the approximations R_M, R_H^f, and R_{MTV}, which were obtained in the foregoing as particular limiting cases of the Hogner approximation R_H, can also be obtained as the first-order (linearized) approximations in asymptotic expansions corresponding to the assumptions that the ship is "thin", "flat", and "slender", respectively, by performing systematic perturbation analyses starting from the usual differential formulation of the problem of steady motion of a ship, as it is well known. In other words, the "perturbation approximations" R_M, R_H^f, and R_{MTV}, are embodied into, indeed are particular limiting cases of, the Hogner approximation R_H, which itself corresponds to the "fine-ship limit" of the "zeroth approximation" R_0 obtained in the present study.

While the neglect of the line integral in formula (10) for the zeroth approximation $\Omega_0(\theta)$ can be justified in the case of a "fine" ship, there is no a priori reason for neglecting this line integral in the case of a ship with a blunt bow or/and stern. The line integral is also particularly significant at low values of the Froude number; specifically, the line integral causes a drastic reduction in the value of the wave resistance at low Froude number, as it will now be shown. For this purpose, we begin by considering the surface integral in formula (10), that is the Hogner "fine-ship approximation" $\Omega_H(\theta)$ given by formula (11). It is convenient here to introduce

the coordinates $\vec{\xi}(\xi,\eta,\zeta)$ which are rendered dimensionless in terms of some characteristic dimension of the ship, say its length L, i.e. we have $\vec{\xi} \equiv \vec{X}/L$ where $\vec{X}$ is dimensional. By using the relation $\vec{x} \equiv \vec{X}g/U^2 \equiv \vec{\xi}/F^2$, where $F \equiv U/(gL)^{1/2}$ is the Froude number, into equation (11), we obtain

$$\Omega_H(\theta) = F^{-4} \int_h e^{F^{-2}[\zeta - i(\xi\cos\theta + \eta\sin\theta)]\sec^2\theta} \nu(\vec{\xi})d\alpha(\vec{\xi}) \quad , \tag{15}$$

where $d\alpha \equiv F^4 da$ is the differential element of area of the surface (h) in terms of the dimensionless variables $\vec{\xi}$.

The major contribution, as $F \to 0$, to the surface integral (15) stems from the immediate vicinity of the "waterline" (c), due to the rapid exponential decay of the factor $\exp(F^{-2}\zeta\sec^2\theta)$ as $F \to 0$. For simplicity, we will restrict our attention to the particular, but fairly common, case when the (fictitious) hull surface (h) intersects the plane $z = 0$ orthogonally, that is when (h) is vertical sided in the vicinity of the waterline. The differential element of area $d\alpha$ of the surface (h) in the immediate neighborhood of the waterline (c) may then be expressed in the form $d\alpha = d\sigma d\zeta$, where $d\sigma \equiv F^2 ds$ is the differential element of arc length along (c). In the low-Froude-number limit, the surface integral (15) may then be approximated as follows

$$\Omega_H(\theta) \sim F^{-4} \int_{\zeta_*}^{0} d\zeta\, e^{F^{-2}\zeta\sec^2\theta} \oint_c e^{-iF^{-2}(\xi\cos\theta + \eta\sin\theta)\sec^2\theta} \nu(\sigma)d\sigma \quad ,$$

where the undefined lower limit of integration ζ_* in the first integral may actually be taken as $(-\infty)$ since the term $\exp(F^{-2}\zeta_*\sec^2\theta)$ is "exponentially small" as $F \to 0$. We may finally obtain

$$\Omega_H(\theta) \sim F^{-2} \cos^2\theta \oint_c e^{-iF^{-2}(\xi\cos\theta + \eta\sin\theta)\sec^2\theta} \mu(\sigma)d\sigma \quad \text{as } F \to 0 \tag{16}$$

since we have $\nu(\sigma) \equiv \mu(\sigma)$ in the present case when (h) intersects the plane $z = 0$ orthogonally.

By combining the line integral in formula (10) and the line integral (16), which is a "low-Froude-number asymptotic approximation" to the surface integral in formula (10), we may obtain the following low-Froude-number asymptotic approximation to the "zeroth approximation" $\Omega_0(\theta)$

$$\Omega_0(\theta) \sim F^{-2} \oint_c e^{-iF^{-2}(\xi\cos\theta + \eta\sin\theta)\sec^2\theta} (\cos^2\theta - \mu^2)\mu d\sigma \quad \text{as } F \to 0 \ . \qquad (17)$$

In the low-Froude-number limit, the exponential factor in the above integral may be seen to be rapidly oscillatory, so that the major contribution to this integral as $F \to 0$ stems from the "points of stationary phase", which are defined by the equation $\xi'\cos\theta + \eta'\sin\theta = 0$, where $\xi' \equiv d\xi/d\sigma$ and $\eta' \equiv d\eta/d\sigma$. At a point of stationary phase we then have $\tan\theta = -\xi'/\eta' = -d\xi/d\eta = -dx/dy$, so that we may obtain $\cos^2\theta = 1/(1 + \tan^2\theta) = dy^2/(dx^2 + dy^2) = (dy/ds)^2$, and the factor $\cos^2\theta - \mu^2$ in the integral (17) vanishes, since we have $\mu \equiv \vec{n}' \cdot \vec{i} \equiv -dy/ds$. It may therefore be seen that, in the low-Froude-number limit, the major contributions of the surface integral and of the line integral in formula (10) for the "zeroth approximation" $\Omega_0(\theta)$ exactly cancel out each other, which evidently results in a drastic reduction in the value of the wave resistance R.

In summary, the main result presented in this section is the "zeroth approximation" $\Omega_0(\theta)$ to the "Kochin free-wave spectrum function" $\Omega(\theta)$. This new approximation Ω_0, which is given by formula (10), may be regarded as a generalization of approximations obtained previously by Michell in 1898, Hogner in 1932, and Maruo, Tuck, and Vossers in the early 1960's; more precisely, these known approximations correspond to particular limiting cases of the "zeroth approximation" Ω_0, as it was shown explicitly. In particular, the zeroth approximation Ω_0 differs from the Hogner "fine-ship approximation" Ω_H given by formula (11) by a line integral around the ship "waterline". While this line integral is small compared with the Hogner surface integral in the "fine-ship limit", that is as the angle α between the "waterline" (c) and the x axis vanishes, and would indeed be ignored in a systematic "fine-(or thin-) ship perturbation analysis", the line integral is actually equal to the Hogner surface integral, at least to first order, in the "low-Froude-number limit", that is as the Froude number F vanishes, as it was just shown. It may therefore be seen that we have

$$\lim_{F \to 0} R_0(F, \alpha = 0) \neq \lim_{\alpha \to 0} R_0(F = 0, \alpha) \ ,$$

that is the limit $\alpha \to 0$, $F \to 0$ is not a uniform limit. This interesting property clearly is related to, indeed is at the origin of, the "low-Froude-number non-uniformity" of the classical "thin-ship perturbation theory".

4. <u>The first approximation</u> R_1

After the "zeroth approximation" $\Omega_0(\theta)$, R_0 — which corresponds to merely taking ϕ as zero in formula (7) for the Kochin free-wave spectrum function $\Omega(\theta)$ — an evident next level of approximation to the Kochin spectrum function $\Omega(\theta)$ is obtained by using the initial potential ϕ_I given by formula (2) as approximation to the disturbance velocity potential ϕ in the ship "near field" in formula (7) for $\Omega(\theta)$. This yields the "first approximation" $\Omega_1(\theta)$, which thus is given by

$$\Omega_1(\theta) = \Omega_0(\theta) + \int_h \phi^I E_n \, da + \oint_c [E^0(\sigma\phi_s^I + \tau\phi_t^I) - \phi^I E_x^0]\mu ds +$$
$$+ \int_f E^0 q_{nl}^I \, dxdy + \int_h E q_{hf}^I \, da - \oint_c E^0 q_{hf}^I \, \nu\mu ds \quad , \tag{18}$$

where equation (10) was used, and the notation $\phi^I \equiv \phi_I$ was used for convenience; the "nonlinear free-surface flux" q_{nl}^I is given by formula (3a), and the "hull form flux" q_{hf}^I is defined by equation (1b) with ϕ replaced by ϕ_I. It will be noted that the various correction terms for "real-fluid effects" q_{bl}, q_{wb}, q_s, and q_w in formula (7) were ignored in formula (18) for the first approximation $\Omega_1(\theta)$, although these correction terms could in principle be included in this first approximation.

An interesting alternative expression for the first approximation $\Omega_1(\theta)$ may be obtained by applying a common Green identity to the functions E and ϕ_I in the domain (d_i) "inside the ship", that is the domain bounded by the fictitious hull surface (h) and the portion (f_i) of the plane $z = 0$ located inside the intersection curve (c) of the surface (h) with the plane $z = 0$. We begin by deriving a preliminary relation valid for an arbitrary function $\psi(\vec{x})$ verifying the Laplace equation $\nabla^2\psi = 0$ in the "interior domain" (d_i) defined above and the Kelvin boundary condition $\psi_z + \psi_{xx} = 0$ on (f_i). By applying a common Green identity to this function $\psi(\vec{x})$ and the function $E(\vec{x};\theta)$ defined by equation (7a), which also verifies the Laplace equation $\nabla^2 E = 0$ in the "interior domain" (d_i), we may obtain

$$\int_h (\psi E_n - E\psi_n) da = \int_{f_i} (\psi E_z - E\psi_z) dxdy \quad ,$$

where the usual notation $E_n \equiv \nabla E \cdot \vec{n}$ and $\psi_n \equiv \nabla\psi \cdot \vec{n}$ was used. From the assumed condition $\psi_z + \psi_{xx} = 0$ for $\vec{x}$ on (f_i) and the relation $E_z + E_{xx} = 0$, which may readily be verified from equation (7a), we have $\psi E_z - E\psi_z = E\psi_{xx} - \psi E_{xx} = (E\psi_x - \psi E_x)_x$, so that we may obtain

$$\int_h (\psi E_n - E\psi_n)da = \int_{f_i} (E\psi_x - \psi E_x)_x \, dxdy \quad .$$

By virtue of a well-known Green identity, we may then obtain

$$\int_h (\psi E_n - E\psi_n)da = \oint_c (E\psi_x - \psi E_x)dy = -\oint_c (E\psi_x - \psi E_x)\mu ds \quad ,$$

in which we used the relation $dy = -\mu ds$, where $\mu \equiv \vec{n}'\cdot\vec{i}$ as it was defined previously in connection with the integral equation (1). Now, we have $\psi_x = \psi_s \vec{s}\cdot\vec{i} + \psi_t \vec{t}\cdot\vec{i} + \psi_n \vec{n}\cdot\vec{i} \equiv \psi_s \sigma + \psi_t \tau + \psi_n \nu$ since $\sigma \equiv \vec{s}\cdot\vec{i}$, $\tau \equiv \vec{t}\cdot\vec{i}$, and $\nu \equiv \vec{n}\cdot\vec{i}$ by definition [the symbols $\vec{i}$, $\vec{s}$, $\vec{t}$, $\vec{n}$, σ , τ , and ν have been defined previously in connection with the integral equation (1)], so that we may finally obtain

$$\int_h \psi E_n da + \oint_c [E^0(\sigma\psi_s + \tau\psi_t) - \psi E^0_x]\mu ds = \int_h E\psi^i_n da - \oint_c E^0\psi^i_n \nu\mu ds \quad , \tag{19}$$

where the notation $E^0 \equiv E(x,y,z{=}0;\theta)$ was used in accordance with equation (7b), and the superscript i in the symbol ψ^i_n is meant to clearly indicate that the normal derivative ψ_n is (evidently) to be evaluated on the "interior side" of the surface (h), which is important in the case when the normal derivative ψ_n is discontinuous across the surface (h).

By using equation (19), with the function ψ taken as the initial potential ϕ_I [which clearly verifies the Laplace equation $\nabla^2\phi_I = 0$ in (d_i) and the Kelvin condition $\phi^I_z + \phi^I_{xx} = 0$ on (f_i)], into formula (18), we may obtain the following alternative expression for the first approximation $\Omega_1(\theta)$

$$\Omega_1(\theta) = \Omega_0(\theta) + \int_h E({}^i\phi^I_n + q^I_{hf})da - \oint_c E^0({}^i\phi^I_n + q^I_{hf})\nu\mu ds + \int_f E^0 q^I_{n1} \, dxdy \ , \tag{20}$$

where the integrals over the fictitious hull surface (h) and the line integrals along the "waterline" (c) have been grouped together. From equation (1b) — with q_{hf} and ϕ replaced by q^I_{hf} and ϕ^I , respectively — we have

$${}^i\phi^I_n + q^I_{hf} = (\nabla\phi_I)_{h_i}\cdot\vec{n} + (\vec{i} + \nabla\phi_I)_H\cdot\vec{N} - (\vec{i} + \nabla\phi_I)_{h_e}\cdot\vec{n} \ ,$$

where the symbols h_i and h_e refer to the interior and exterior sides of the surface

(h), respectively. However, we have the relation

$$(\nabla\phi_I)_{h_i} \cdot \vec{n} - (\nabla\phi_I)_{h_e} \cdot \vec{n} = \vec{i} \cdot \vec{n} \tag{21}$$

as a result of the fact that the initial potential ϕ_I involves a distribution of sources of density $\nu \equiv \vec{n} \cdot \vec{i}$ on the surface (h), as it may be seen from formula (2). We may then obtain

$$^{i}\phi_n^I + q_{hf}^I = (\vec{i} + \nabla\phi_I)_H \cdot \vec{N} \equiv (\nu + \phi_n^I)_H \quad ,$$

and formula (20) may finally be expressed in the form

$$\boxed{\Omega_1(\theta) = \Omega_0(\theta) + \int_h E(\nu + \phi_n^I)_H da - \oint_c E^0(\nu + \phi_n^I)_H \nu\mu ds + \int_f E^0 q_{nl}^I dxdy} \quad . \tag{22}$$

It is interesting to compare formulas (2) and (3) for the initial and second approximations ϕ_I and ϕ_2 to the disturbance velocity potential ϕ and formulas (10) and (22) for the zeroth and first approximations Ω_0 and Ω_1 to the Kochin free-wave spectrum function Ω; it may be seen that formulas (2) and (10) and formulas (3) and (22) directly correspond to each other with the substitution $G \longleftrightarrow E$. Numerical evaluation of the first approximation $\Omega_1(\theta)$ defined by formula (22) may be divided into five basic steps, as follows: (i) evaluate the zeroth approximation $\Omega_0(\theta)$ defined by formula (10) for some "fictitious" hull surface (h), which may — but need not — be taken as the wetted hull of the ship in position of rest, (ii) evaluate the initial potential ϕ_I defined by formula (2), (iii) determine the sinkage and trim experienced by the ship and the position of the "real" ship hull surface (H) corresponding to the approximation ϕ_I, (iv) evaluate the fluid flux $(\nu + \phi_n^I)_H \equiv [\vec{i} + (\nabla\phi_I)_H] \cdot \vec{N}$ across the ship hull (H) and the nonlinear free-surface correction flux q_{nl}^I given by formula (3a), and finally (v) evaluate the three integrals shown in formula (22). The first approximation R_1 to the wave resistance R may then be determined by evaluating the Havelock wave resistance integral (9) with $\Omega(\theta)$ replaced by the first approximation $\Omega_1(\theta)$. The computational task involved in the practical implementation of the above-described successive steps admittedly is rather considerable, but it ought however to be well within present-day calculation capabilities.

5. The second approximation R_2

As it was noted previously, the zeroth and first approximations $\Omega_0(\theta)$ and $\Omega_1(\theta)$ may be regarded as the first two approximations in a sequence of iterative approximations $\Omega_k(\theta)$, $k \geq 0$, associated with the iterative approximations $\phi_0 \equiv 0, \phi_I, \phi_2, \ldots$ to the solution ϕ of the integral equation (1) for the disturbance velocity potential in the ship "near field". The second approximation $\Omega_2(\theta)$ corresponding to the second approximation ϕ_2 given by formula (3) will now be derived. By replacing ϕ by ϕ_2 — which will be denoted by $\phi^{(2)}$ for convenience — in formula (7) for the Kochin free-wave spectrum function $\Omega(\theta)$, we readily obtain the following expression for the second approximation $\Omega_2(\theta)$:

$$\Omega_2(\theta) = \Omega_0(\theta) + \int_h \phi^{(2)} E_n da + \oint_c [E^0(\sigma\phi_s^{(2)} + \tau\phi_t^{(2)}) - \phi^{(2)} E_x^0]\mu ds + \\ + \int_f E^0 q_{nl}^{(2)} dxdy + \int_h E q_{hf}^{(2)} da - \oint_c E^0 q_{hf}^{(2)} \nu\mu ds \quad , \tag{23}$$

where formula (10) was used, the ad-hoc correction terms q_{bl}, q_{wb}, q_s, and q_w for "real-fluid effects" were ignored for simplicity, and the terms $q_{nl}^{(2)}$ and $q_{hf}^{(2)}$ are defined by equations (1a) and (1b) with ϕ replaced by $\phi^{(2)}$.

It may easily be seen that the potential $\phi^{(2)} \equiv \phi_2$ defined by formula (3) verifies the Laplace equation $\nabla^2\phi^{(2)} = 0$ in the domain (d_i) inside the fictitious hull surface (h) and the Kelvin condition $\phi_z^{(2)} + \phi_{xx}^{(2)} = 0$ on the portion (f_i) of the plane $z = 0$ located inside (h), so that equation (19) — with ψ replaced by $\phi^{(2)}$ — may be used. Expression (23) for the second approximation $\Omega_2(\theta)$ then becomes

$$\Omega_2(\theta) = \Omega_0(\theta) + \int_h E({}^i\phi_n^{(2)} + q_{hf}^{(2)})da - \oint_c E^0({}^i\phi_n^{(2)} + q_{hf}^{(2)})\nu\mu ds + \int_f E^0 q_{nl}^{(2)} dxdy \ , \tag{24}$$

where the integrals over the fictitious hull surface (h) and the line integrals along the "waterline" (c) have been grouped together, and the symbol ${}^i\phi_n^{(2)}$ is meant for the normal derivative $\phi_n^{(2)} \equiv \nabla\phi_2 \cdot \vec{n}$ evaluated on the interior side of the surface (h).

From equation (1b) — with q_{hf} and ϕ replaced by $q_{hf}^{(2)}$ and $\phi^{(2)} \equiv \phi_2$, respectively — we have

$${}^i\phi_n^{(2)} + q_{hf}^{(2)} = (\nabla\phi_2)_{h_i} \cdot \vec{n} + (\vec{i} + \nabla\phi_2)_H \cdot \vec{N} - (\vec{i} + \nabla\phi_2)_{h_e} \cdot \vec{n} \quad ,$$

where the symbols h_i and h_e refer to the interior and exterior sides of the surface (h), respectively, as it was defined in the previous section. However, we have the relation

$$(\nabla\phi_2)_{h_i} \cdot \vec{n} - (\nabla\phi_2)_{h_e} \cdot \vec{n} = \vec{i} \cdot \vec{n} + (\nu + \phi_n^I)_H$$

as a result of the fact that the potential ϕ_2 involves a distribution of sources of density $\nu + (\nu + \phi_n^I)_H \equiv \vec{n} \cdot \vec{i} + (\vec{i} + \nabla\phi_I)_H \cdot \vec{N}$ on the fictitious hull surface (h), as it may be seen from formulas (3) and (2). We may then obtain

$$^i\phi_n^{(2)} + q_{hf}^{(2)} = (\nu + \phi_n^I)_H + (\nu + \phi_n^{(2)})_H \quad . \tag{25}$$

We may express formula (1a) — with q_{n1} and ϕ replaced by $q_{n1}^{(2)}$ and $\phi^{(2)}$ — in the form

$$q_{n1}^{(2)} = \pi^{(2)} - (\phi_z^{(2)} + \phi_{xx}^{(2)})_{z=0} \quad ,$$

where the term $\pi^{(2)}$ thus is defined as

$$\pi^{(2)} = \left[\phi_z^{(2)} + \phi_{xx}^{(2)} + (|\nabla\phi^{(2)}|^2)_x + \tfrac{1}{2}\nabla\phi^{(2)}\cdot\nabla|\nabla\phi^{(2)}|^2\right]_{z = -\phi_x^{(2)} - \frac{1}{2}|\nabla\phi^{(2)}|^2} \quad , \tag{26}$$

However, we have the relation

$$(\phi_z^{(2)} + \phi_{xx}^{(2)})_{z=0} = -q_{n1}^I \text{ for } (x,y) \text{ on } (f) \tag{27}$$

as a result of the fact that the potential ϕ_2 involves a "free-surface flux" distribution of strength q_{n1}^I on (f), as it may be seen from formula (3). We may then obtain

$$q_{n1}^{(2)} = q_{n1}^I + \pi^{(2)} \quad . \tag{28}$$

By substituting equations (25) and (28) into expression (24) for $\Omega_2(\theta)$, and by using expression (22) for $\Omega_1(\theta)$, we may finally obtain the following alternative expression for the second approximation $\Omega_2(\theta)$

$$\boxed{\Omega_2(\theta) = \Omega_1(\theta) + \int_h E(\nu + \phi_n^{(2)})_H da - \oint_c E^0(\nu + \phi_n^{(2)})_H \nu\mu ds + \int_f E^0\pi^{(2)} dxdy} \quad , \tag{29}$$

which is identical in form to expression (22) for the first approximation $\Omega_1(\theta)$.

6. Discussion

Higher approximations R_k, $k \geq 3$, to the wave resistance R can easily be defined in principle. Specifically, it can be shown that the approximations $\phi_I \equiv \phi^{(1)}$ and $\phi_2 \equiv \phi^{(2)}$ to the disturbance velocity potential ϕ defined by formulas (2) and (3) actually correspond to the first two approximations in the sequence of iterative approximations $\phi^{(k)}$ defined by the "zeroth approximation"

$$\phi^{(0)} \equiv 0 \tag{30a}$$

and the recurrence relation

$$\phi^{(k+1)}(\vec{x}_0) = \phi^{(k)}(\vec{x}_0) + \int_h G(\nu+\phi_n^{(k)})_H da - \oint_c G(\nu+\phi_n^{(k)})_H \nu\mu ds + \int_f G\pi^{(k)} dxdy \;, \; k\geq 0, \tag{30b}$$

where the nonlinear free-surface correction flux $\pi^{(k)}$ is given by

$$\pi^{(k)} = \left[\phi_z^{(k)} + \phi_{xx}^{(k)} + (|\nabla\phi^{(k)}|^2)_x + \tfrac{1}{2}\nabla\phi^{(k)}\cdot\nabla|\nabla\phi^{(k)}|^2\right]_{z=-\phi_x^{(k)} - \frac{1}{2}|\nabla\phi^{(k)}|^2} \;. \tag{30c}$$

The approximations Ω_0, Ω_1, and Ω_2 to the Kochin free-wave spectrum function $\Omega(\theta)$ similarly correspond to the first three approximations in the sequence of iterative approximations $\Omega_k(\theta)$ defined by the "zeroth approximation"

$$\Omega_0(\theta) = \int_h E\nu da - \oint_c E^0 \nu^2 \mu ds \tag{31a}$$

associated with the "zeroth approximation" $\phi^{(0)} \equiv 0$ to ϕ , and the recurrence relation

$$\Omega_k(\theta) = \Omega_{k-1}(\theta) + \int_h E(\nu+\phi_n^{(k)})_H da - \oint_c E^0(\nu+\phi_n^{(k)})_H \nu\mu ds + \int_f E^0\pi^{(k)} dxdy, \; k\geq 1 \;. \tag{31b}$$

The Havelock wave-resistance formula (9) — with $\Omega(\theta)$ and R replaced by $\Omega_k(\theta)$ and R_k, respectively — then readily defines the sequence of iterative approximations R_k, $k\geq 0$, to the wave resistance R. To the above equations, we should actually add equations for determining the lift and moment and the resulting sinkage and trim experienced by the ship. It may also be noted that although the ad-hoc correction terms for "real-fluid effects" q_{b1}, q_{wb}, q_s, and q_w in the integral equation (1) and formula (7) were ignored for simplicity in the iterative approximations $\phi^{(k)}$ and Ω_k defined by equations (30) and (31), these correction terms could be incorporated in principle. The iterative

scheme defined by the recurrence relations (30b) and (31b) can be pursued in principle (assuming convergence) until the "hull flux" $(\nu + \phi_n^{(k)})_H \equiv [\vec{i} + (\nabla\phi^{(k)})_H] \cdot \vec{N}$ and the "nonlinear free-surface flux" $\pi^{(k)}$ are sufficiently small, that is until the boundary conditions at the ship hull surface and at the free surface — which may indeed be expressed in the form $[\vec{i} + (\nabla\phi)_H] \cdot \vec{N} = 0$ and $\pi = 0$, respectively — are verified within desired accuracy.

Whereas in principle the above-defined approximations ϕ_k, Ω_k, R_k can readily be determined by using the recurrence relations (30b) and (31b) and the Havelock wave resistance formula (9), the enormous computational task involved in the actual numerical evaluation of these higher approximations drastically restrict the feasibility of the approach in practice. As a matter of fact, evaluation of the second approximation ϕ_2, Ω_2, R_2 may already be too considerable a task in practice, and the first approximation ϕ_I, Ω_1, R_1 defined by formulas (2), (10), (22), and (9) may well be the point up to which it is actually feasible to pursue the iterative scheme defined by the recurrence relations (30b) and (31b). For practical purposes, the first approximation ϕ_I, Ω_1, R_1 may indeed be regarded as the main result of the present study.

Alternative methods of refining the first approximation ϕ_I, Ω_1, R_1 that are simpler to implement numerically than the continuation of the above-described iterative scheme beyond the first approximation may however be envisioned. Two such methods may indeed be mentioned here briefly (as they will be described in detail in Parts 4 and 5 of the present potential theory of steady motion of ships). A very simple modification of the first approximation ϕ_I which seems likely to improve somewhat this approximation is given by the "modified initial potential" $\phi_I' \equiv k\phi_I$, where k is a constant which can be determined from a straightforward consideration of potential flow about a triaxial ellipsoid, with main dimensions equal to that of the given ship, in the "zero-Froude-number approximation" (in which the free surface is replaced by a rigid wall), as it will now be explained. The "zero-Froude-number integral equation", that is the integral equation for the disturbance velocity potential, φ say, in the "zero-Froude-number approximation", takes the form

$$\varphi(\vec{x}_0) = \varphi_I(\vec{x}_0) + \int_h [\varphi(\vec{x}) - \varphi(\vec{x}_0)] G_n^0(\vec{x}_0,\vec{x}) da(\vec{x}) \quad , \tag{32}$$

where the function $\varphi_I(\vec{x}_0)$ is the "zero-Froude-number initial potential" defined as

$$\varphi_I(\vec{x}_0) = \int_h G^0(\vec{x}_0,\vec{x}) \nu(\vec{x}) da(\vec{x}) \quad , \tag{32a}$$

and $G^0(\vec{x}_0,\vec{x})$ is the "zero-Froude-number Green function" given by $4\pi G^0(\vec{x}_0,\vec{x})=-1/|\vec{x}-\vec{x}_0|$ $-1/|\vec{x}-\vec{x}_1|$, with $\vec{x}_1$ representing the mirror image of the point $\vec{x}_0$ with respect to the plane z=0. Clearly, the integral equation (32) corresponds to the integral equation (1), with the various ad-hoc correction terms for "real-fluid effects" being ignored, while the initial approximation φ_I given by formula (32a) corresponds to the approximation ϕ_I given by formula (2). It is interesting that in the case of a triaxial ellipsoid, say with semiaxes of length a, b, and c, the "modified initial potential" φ_I' defined as $\varphi_I' = k\varphi_I$ — where k is some given function of a, b, and c, or more precisely of the ratios b/a and c/b — actually is the exact potential φ. This remarkable result suggests that the "modified initial potential" $\phi_I' = k\phi_I$, where k is the above-mentioned function of the ratios b/a and c/b (that is the beam/length and draft/beam ratios), may provide some improvement of the original initial potential ϕ_I given by formula (2). The "modified initial potential" ϕ_I', and the associated modified approximations Ω_1' and R_1' will be examined in some detail in Part 5 of this study. One may also seek to improve the initial potential ϕ_I, and consequently the approximations Ω_1 and R_1, by taking advantage of the fact that the "fictitious" hull surface (h) in formula (2) may be chosen at will, at least to a certain extent. An obvious way of exploiting this arbitrariness in the choice of the surface (h) is provided by the "method of coordinates straining", which was indeed used previously for a quite analogous purpose by Noblesse and Dagan [10]. This modification of the analysis developed in [10] will also be presented in Part 5.

Besides seeking to <u>refine</u> the first approximation ϕ_I, Ω_1, R_1 we may also seek to <u>simplify</u> this first approximation, particularly from the point of view of numerical implementation. Two such simplifications may also be briefly mentioned here. A notable simplification of the above-defined first approximation may be achieved in the case of ships operating at fairly low values of the Froude number, for which the "zero-Froude-number initial potential" φ_I given by formula (32a), or better yet the modified potential $\varphi_I' \equiv k\varphi_I$, may be used as an approximation to the potential ϕ in the ship "near field" in formula (7) for the Kochin free-wave spectrum function $\Omega(\theta)$, thereby defining a "low-Froude-number slender-ship approximation". This approximation will be examined in detail in Part 4 of this study, which will be concerned with "low-Froude-number approximations". A different sort of simplification of the first approximation ϕ_I, Ω_1, R_1 can be achieved by again exploiting the fact that the "fictitious" hull surface (h) may, to a certain extent, be chosen at will. Specifically, important simplifications can be achieved by assuming the framelines of the fictitious hull (h) to be trapezoidal in shape, as it will be shown explicitly in Part 5.

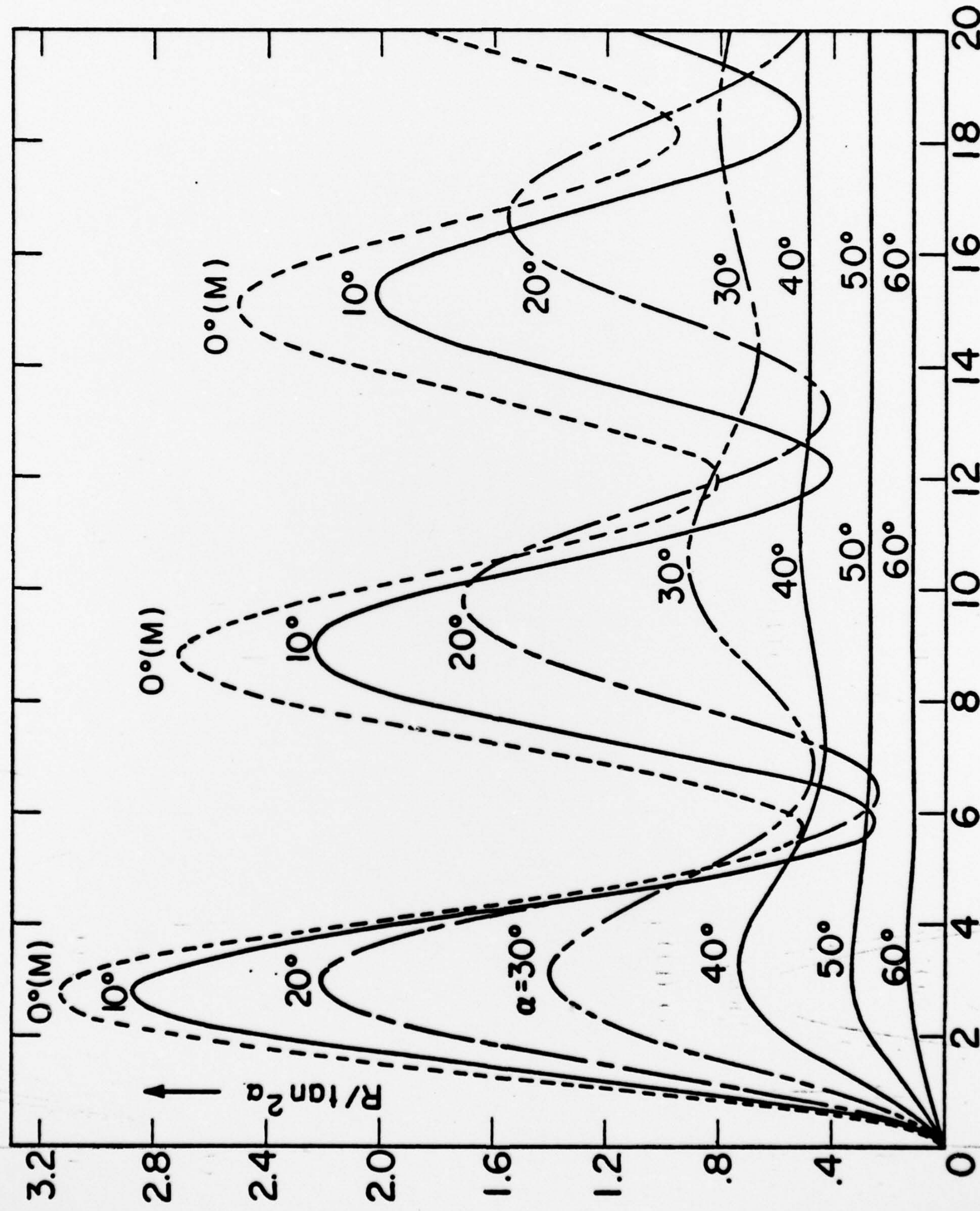
7. Comparison between the zeroth approximation R_0 and the Michell approximation R_M for a wedge-like ship-bow form

A comparison between the classical Michell "thin-ship approximation" R_M and the new "zeroth approximation" R_0 obtained in the previous section is shown in figure 2 for the case of the wedge-like ship-bow form defined by the equations $y = \pm x\tan\alpha$ for $0 \leq x \leq \ell$ and $0 \geq z \geq -\infty$, and $y = \pm \ell\tan\alpha$ for $\ell \leq x \leq \infty$ and $0 \geq z \geq -\infty$. This ship-bow form thus is of infinite draft, and consists of an "entrance wedge" of angle 2α and (dimensionless) length $\ell(\equiv Lg/U^2$ where L is dimensional) continued by two parallel vertical walls extending to infinity downstream. Specifically, figure 2 shows the function $R_0^*(\ell;\alpha)$ for $0 \leq \ell \leq 20$ and for $\alpha = 0°, 10°, 20°, 30°, 40°, 50°$, and $60°$, where R_0^* is defined as $R_0^* \equiv R_0/\tan^2\alpha$, with R_0 dimensionless (we have $R \equiv R'g^2/\rho U^6$ where R' is dimensional). As it was discussed previously, the "zeroth approximation" R_0 reduces to the Michell approximation R_M in the "thin-ship limit", that is as $\alpha \to 0$. This may indeed be verified from figure 2 where the curve corresponding to $\alpha = 0°$ is actually identical to the Michell approximation $R_M^*(\ell) \equiv R_M/\tan^2\alpha \equiv R_0^*(\ell;\alpha = 0°)$, and both the symbols 0° and M (for Michell) are indeed attached to this curve. It will be noted that the Michell approximation $R_M(\ell,\alpha)$ is proportional to $\tan^2\alpha$, that is we have $R_M(\ell,\alpha) = R_M^*(\ell)\tan^2\alpha$, and the curves $R^*(\ell;\alpha)$ therefore collapse into one single curve $R_M^*(\ell)$ in the Michell approximation, while we do have a family of different curves in the "zeroth approximation" R_0^* .

It may be seen from figure 2 that the curves $R_0^*(\ell;\alpha)$ generally lie below the Michell curve $R_M^*(\ell) \equiv R_0^*(\ell;\alpha = 0°)$ and that the differences between the "zeroth approximation" R_0^* and the Michell approximation R_M^* are quite significant even for such relatively small entrance angles as $\alpha = 10°$ and $\alpha = 20°$, and are in fact rather spectacular for larger entrance angles, say for $\alpha \geq 30°$. In particular, the amplitude of the oscillations in the Michell wave-resistance curve are notably reduced in the "zeroth-approximation" even for such relatively small angles as $\alpha = 10°$ and $\alpha = 20°$, while for larger angles, say for $\alpha \geq 30°$, these oscillations are drastically reduced. One may also observe a notable phase shift of the curves $R_0^*(\ell;\alpha)$ with respect to the Michell curve $R_M^*(\ell)$. This phase shift is directed towards the higher values of the "entrance length" ℓ, that is towards the lower values of the Froude number $F \equiv \ell^{-1/2}$, and it may be seen to increase with both α and ℓ.

The trend of the above results for the "zeroth approximation" R_0 is encouraging, and suggests that the present new theory may help explain, and remedy to, some of the typical discrepancies between theoretical and experimental wave-resistance curves. Confirmation of this hope must however await further numerical calculations, and comparisons with experimental results. To this end, numerical calculations for the case of a parabolic strut are currently being undertaken.

Figure 2: Comparison between the zeroth approximation R_0 and the Michell approximation R_M for a wedge-like ship-bow form.



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